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DISCUSSION

N_q factor for pile foundations by Berezantzev

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The author is correct in pointing out that the bearing capacity factors for deep foundations developed by Berezantzev *et al.* (1961), often just referred to as Berezantzev, have been adopted by many engineers in the piling industry. However, the reason for this preference over other more rigorous and justifiable theories is difficult to explain, particularly as the Berezantzev method tends to give more conservative values for N_q and N_γ that are lower than those given by many other theories. Despite the improvement in accuracy of the depth correction factor α_t calculated by the author, there are still some underlying concerns regarding the validity of the resultant N_q values. These are discussed briefly below.

It is important to recognise that the N_q factors presented by Berezantzev comprise two components: a bearing factor B_k, and a depth correction factor α_t. Most references to Berezantzev cover the relationship between B_k and ϕ' (Poulos & Davis, 1980; Fleming *et al*., 1992; Randolph *et al.*, 1994). Apart from Tomlinson (1994), the discusser is unaware of any other significant reference that includes the depth correction factor α_t. This may be due to the fact that α_t is a reduction factor, which implies that the effective end bearing capacity factor $N_q = \alpha_t B_k$ reduces with pile penetration. This is not an intuitive result. It is also in direct conflict with other bearing capacity theories such as those presented by Brinch Hansen (1961), Vesic (1967), or Meyerhof (1975). Nor is it supported by studies based on cavity expansion theories such as Vesic (1972) or Yu & Houlsby (1991/1992). As a result, it is believed that most practitioners who use the Berezantzev bearing capacity factors use just the B_k component and tend to disregard α_t.

Part of the difficulty in justifying the use of the Berezantzev bearing capacity factors is that the 1961 paper concentrates on the development of the reduction factor α_t and does not give any background theory for B_k. It is necessary to go back to the original paper by Berezantzev (1952). Unfortunately, even this background paper does not hold up too well against proper scrutiny. The A_k and B_k factors in the 1952 paper are slightly different from those given in the 1961 paper. Although based on a mathematical framework, it is clear that key functions used to compute the B_k factor have been approximated based on model test results at low stress levels, which may not be applicable to the higher stresses appropriate to the soil zones below the base of deep foundations. Likewise, the theoretical basis for the reduction factor α_t is questionable, and does not allow for any interaction between the pile shaft and the end bearing mechanism.

With regard to the reducing rate of increase in end bearing stress and the apparent threshold or limiting value suggested by a number of authors including Vesic (1970), the technical note presents an attractive approach, resulting in a rapid reduction of N_q with increasing pile penetration. This is illustrated in Fig. 1, where end bearing stresses given in the note are compared with those computed using the Berezantzev B_k factor alone, and with Brinch Hansen (1961) theory, both based on the approach given by Randolph *et al.* (1994) and Bolton (1986/1987). Fig. 1 also includes stresses based on a cavity expansion approach following Yu & Houlsby (1991/1992). Tabulated values for q_b, N_q and ϕ' are given in Table 3.

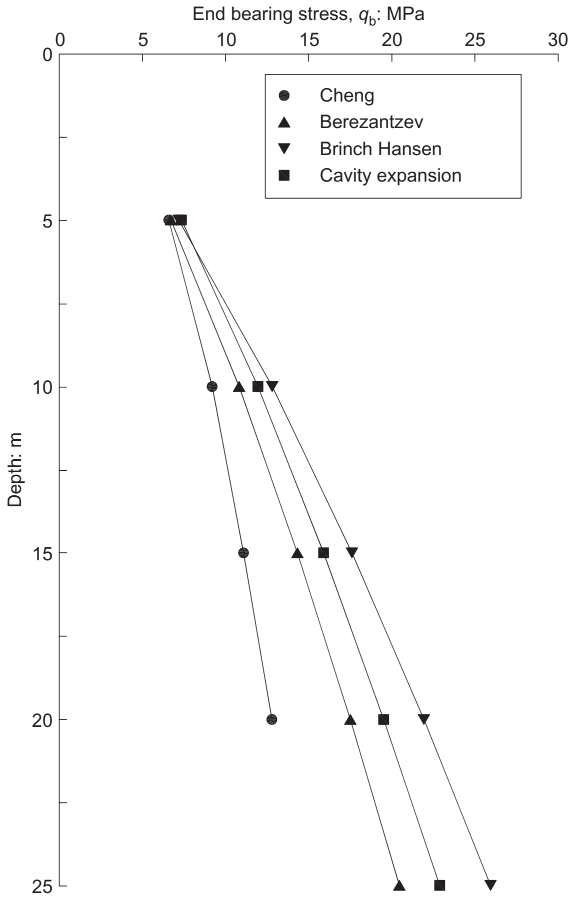

Fig. 1. Comparison of ultimate end bearing pressures

Unfortunately the technical note gives little theoretical justification for use of the reduction factor α_t, and does not give an explanation for the inconsistency with other bearing capacity theories. Many authors have argued that the apparent threshold or limiting end bearing stress is not a real phenomenon, and can be fully explained by a combination of other effects, including suppression of dilation under high confining stresses, soil compressibility, or even pile test interpretation problems. Examples include Bolton (1986/1987), Neely (1990), Kraft (1991) and Randolph *et al.* (1994). There are clearly sufficient alternative rational explanations available in the literature to explain the reducing rate of increase in end bearing stress.

In conclusion, although the improved accuracy of the Berezantzev reduction factor α_t is welcome, the discusser believes that the continued use of a bearing capacity theory that was developed more than 40 years ago is difficult to justify, particularly as it is based on a somewhat questionable theoretical background, and does not implicitly include for the effects of soil compressibility or variation in ϕ'. Alternative methods based on cavity expansion theory seem to be the most promising direction for future research and development. This is a topic area that should perhaps be revisited by researchers, and it would certainly benefit from the application of 21st century computer-based numerical modelling techniques.

Table 3. Friction angle, bearing capacity factor and end bearing stress

Depth: m	Vertical stress: kPa	Friction angle, ϕ': deg				Bearing capacity factor, N_q				End bearing stress, q_b: kPa			
		Cheng	Berezantzev	Brinch Hansen	Cavity expansion	Cheng	Berezantzev	Brinch Hansen	Cavity expansion	Cheng	Berezantzev	Brinch Hansen	Cavity expansion
5	100	34·5	34·4	34·3	32·5	66·0	67·2	71·9	73·6	6600	6717	7190	7356
10	200	33·3	33·1	32·9	31·9	46·0	54·1	64·0	59·7	9200	10 817	12 807	11 946
15	300	32·6	32·3	32·1	31·5	37·0	47·8	58·7	53·0	11 100	14 330	17 616	15 908
20	400	32·1	31·8	31·5	31·2	32·0	43·8	54·9	48·8	12 800	17 515	21 942	19 520
25	500		31·4	31·1	31·0		41·0	51·9	45·8		20 477	25 942	22 894

NOTATION

A_k Berezantzev bearing capacity factor
B_k Berezantzev bearing capacity factor
N_q bearing capacity factor
N_γ bearing capacity factor
q_b end bearing pressure
α_t Berezantzev depth correction factor
ϕ' angle of shearing resistance

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